

Study Guide

Arithmetic Sequences

Recognize Arithmetic Sequences A sequence is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an arithmetic sequence.

Arithmetic Sequence

a numerical pattern that increases or decreases at a constant rate or value called the common difference

Example 1

Determine whether the sequence 1, 3, 5, 7, 9, 11, ... is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since $3 - 1 = 2$, $5 - 3 = 2$, and so on, the common difference is 2.

Since the difference between the terms of 1, 3, 5, 7, 9, 11, ... is constant, this is an arithmetic sequence.

Example 2

Determine whether the sequence 1, 2, 4, 8, 16, 32, ... is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since $2 - 1 = 1$ and $4 - 2 = 2$, there is no common difference.

Since the difference between the terms of 1, 2, 4, 8, 16, 32, ... is not constant, this is not an arithmetic sequence.

Exercises

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

1. $1, 5, 9, 13, 17, \dots$

yes
 $d = 4$

2. $8, 4, 0, -4, -8, \dots$

yes
 $d = -4$

3. $1, 3, 9, 27, 81, \dots$

no

4. $10, 15, 25, 40, 60, \dots$

no

5. $-10, -5, 0, 5, 10, \dots$

yes
 $d = 5$

6. $8, 6, 4, 2, 0, -2, \dots$

yes
 $d = -2$

7. $4, 8, 12, 16, \dots$

yes
 $d = 4$

8. $15, 12, 10, 9, \dots$

no

9. $1.1, 2.1, 3.1, 4.1, 5.1, \dots$

yes
 $d = 1$

10. $8, 7, 6, 5, 4, \dots$

yes
 $d = -1$

11. $0.5, 1.5, 2.5, 3.5, 4.5, \dots$

yes
 $d = 1$

12. $1, 4, 16, 64, \dots$

no

13. $10, 14, 18, 22, \dots$

yes
 $d = 4$

14. $-3, -6, -9, -12, \dots$

yes
 $d = -3$

15. $7, 0, -7, -14, \dots$

yes
 $d = -7$

Study Guide (continued)

Arithmetic Sequences

Write Arithmetic Sequences You can use the common difference of an arithmetic sequence to find the next term of the sequence. Each term after the first term is found by adding the preceding term and the common difference.

Terms of an Arithmetic Sequence	If a_1 is the first term of an arithmetic sequence with common difference d , then the sequence is $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \dots$
nth Term of an Arithmetic Sequence	$a_n = a_1 + (n - 1)d$

Example 1 Find the next three terms of the arithmetic sequence 28, 32, 36, 40, ...

Find the common difference by subtracting successive terms.

$$\begin{array}{cccc} 28 & 32 & 36 & 40 \\ \downarrow & \downarrow & \downarrow & \\ +4 & +4 & +4 & \end{array}$$

The common difference is 4.

Add 4 to the last given term, 40, to get the next term. Continue adding 4 until the next three terms are found.

$$\begin{array}{cccc} 40 & 44 & 48 & 52 \\ \downarrow & \downarrow & \downarrow & \\ +4 & +4 & +4 & \end{array}$$

The next three terms are 44, 48, 52.

Example 2 Write an equation for the n th term of the sequence 12, 15, 18, 21, ...

In this sequence, a_1 is 12. Find the common difference.

$$\begin{array}{cccc} 12 & 15 & 18 & 21 \\ \downarrow & \downarrow & \downarrow & \\ +3 & +3 & +3 & \end{array}$$

The common difference is 3.

Use the formula for the n th term to write an equation.

$$a_n = a_1 + (n - 1)d \quad \text{Formula for the } n\text{th term}$$

$$a_n = 12 + (n - 1)3 \quad a_1 = 12, d = 3$$

$$a_n = 12 + 3n - 3 \quad \text{Distributive Property}$$

$$a_n = 3n + 9 \quad \text{Simplify.}$$

The equation for the n th term is $a_n = 3n + 9$.

Exercises

Warm Up 1-6

Find the next three terms of each arithmetic sequence.

1. 9, 13, 17, 21, 25, ... $d=4$
29, 33, 37

2. 4, 0, -4, -8, -12, ... $d=-4$
-16, -20, -24

3. 29, 35, 41, 47, ... $d=6$
53, 59, 65

4. -10, -5, 0, 5, ... $d=5$
10, 15, 20

5. 2.5, 5, 7.5, 10, ... $d=2.5$
12.5, 15, 17.5

6. 3.1, 4.1, 5.1, 6.1, ... $d=1$
7.1, 8.1, 9.1

Find the n th term of each arithmetic sequence described.

7. $a_1 = 6, d = 3, n = 10$

8. $a_1 = -2, d = -3, n = 8$

9. $a_1 = 1, d = -5, n = 20$

10. $a_1 = -3, d = -2, n = 50$

11. $a_1 = -12, d = 4, n = 20$

12. $a_1 = 1, d = \frac{1}{2}, n = 11$

Write an equation for the n th term of the arithmetic sequence.

13. 1, 3, 5, 7, ...

14. -1, -4, -7, -10, ...

15. -4, -9, -14, -19, ...

$$7. a_n = 6 + (n-1)3$$

$$a_n = 3n + 3$$

$$a_{10} = 3(10) + 3$$

$$a_{10} = 33$$

$$8. a_n = -2 + (n-1)(-3)$$

$$a_n = -3n + 1$$

$$a_8 = -3(8) + 1$$

$$a_8 = -23$$

$$9. a_n = 1 + (n-1)(-5)$$

$$a_n = -5n + 6$$

$$a_{25} = -5(25) + 6$$

$$a_{25} = -94$$

$$10. a_n = -3 + (n-1)(-2)$$

$$a_n = -2n - 1$$

$$a_{50} = -2(50) - 1$$

$$a_{50} = -101$$

$$11. a_n = -12 + (n-1)4$$

$$a_n = 4n - 16$$

$$a_{20} = 4(20) - 16$$

$$a_{20} = 64$$

$$12. a_n = 1 + (n-1)\frac{1}{2}$$

$$a_n = \frac{1}{2}n + \frac{1}{2}$$

$$a_{11} = \frac{1}{2}(11) + \frac{1}{2}$$

$$a_{11} = 6$$

$$13. a_1 = 1 \quad d = 2$$

$$a_n = 1 + (n-1)2$$

$$a_n = 2n - 1$$

$$14. a_1 = -1 \quad d = -3$$

$$a_n = -1 + (n-1)(-3)$$

$$a_n = -3n + 2$$

$$15. a_1 = -4 \quad d = -5$$

$$a_n = -4 + (n-1)(-5)$$

$$a_n = -5n + 1$$

$$\begin{aligned}
(1) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(2) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(3) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(4) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(5) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(6) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(7) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(8) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$

$$\begin{aligned}
(9) \quad & (1) - (1) + 2 = 2 \quad (1) - (1) + 2 = 2 \\
& 1 + 2 = 3 \\
& 1 + (2) = 3 \\
& 2 = 3
\end{aligned}$$