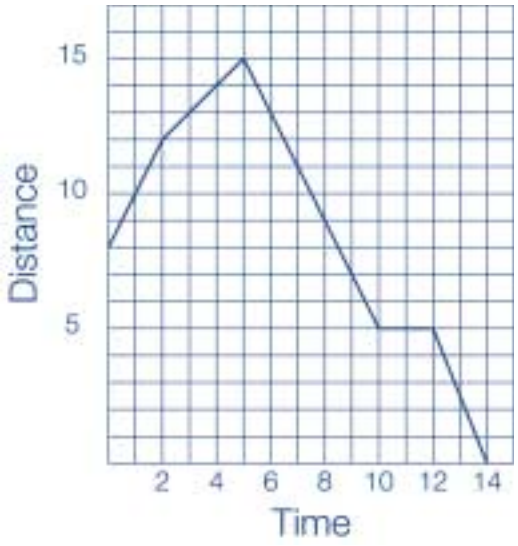
Distance and Time

The graph below represents distance as a function of time.



Create a situation this graph might represent. Choose appropriate units for time and distance. Describe your situation in detail.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Describe some situations in which the variables could be distance and time.
- For the situation you choose, what are reasonable units for time and distance?
- Describe how to break the graph up into phases.
- What do the graphs in the phases show you in terms of the function increasing, decreasing, or being constant? What does this mean in your situation?
- Can you determine the slope of the graph in each phase? What will this mean in your situation?
- What are the *x*- and *y*-intercepts for the graph? What do they mean in the situation you chose?

Sample Solution:

A wind blows a leaf off a tree branch about 8 feet above the ground. The wind swirls the leaf upwards at a constant rate of 2 feet per second for one second. Now the leaf is 12 feet above the ground. The wind slows down. The leaf swirls upwards at a constant rate of 1 foot per second, reaching a height of 15 feet. From 5 seconds to 10 seconds the wind subsides. The leaf falls at a steady rate of 2 feet per second to 5 feet above the ground and lands on another tree branch. It stays on the branch for two seconds until a slight breeze catches the leaf and it falls to the ground at a steady rate of 5 feet per 2 seconds. The leaf's journey from tree branch to ground lasted 14 seconds.

Extension Questions:

- Consider the phases: ① 0 to 2, ② 2 to 5, ③ 5 to 10, ④ 10 to 12, and ⑤ 12 to 14. In which phases is the function increasing? Decreasing? Constant? What does this tell you about the slope of each phase?

The function is increasing in phases ① and ②, so the line slopes upward, and the rate of travel is positive.

The function is decreasing in phases ③ and ⑤, so the line slopes downward, and the rate is negative.

In phase ④ the distance remains constant; the line is horizontal; the rate of travel is 0 feet per second.

- What does the slope of each phase mean in the situation you created?

Since it equals change in distance between two points divided by change in time for the two points, it represents velocity or speed and direction at which the object travels.

- How would you interpret this graph if the dependent quantity was velocity instead of distance?

In Phase ①, an object is moving with constant positive speed. In phase ②, the velocity is still steady but slower than before. In phase ③, the distance is now decreasing at a steady rate; the velocity is a negative number. In phase ④, the velocity is zero because the object does not move. Finally, in phase ⑤, the distance decreases; the velocity is negative.

- If the first phase on the graph had been from the point (0,12) to the point (2,12), how would that change your description of the graph?

The object was still for the first two seconds before it started to move because the distance remained constant.

- Take the information from the graph and create a graph of the velocity of the object as a function of time. The velocity is the speed at which the object traveled.

The velocity can be found for each phase.

Time Interval	Velocity
0 to 2	2
2 to 5	1
5 to 10	-2
10 to 12	0
12 to 14	-2.5

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns
3 Interpreting Graphs
3.1 Interpreting Distance versus Time Graphs

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

The graph is a series of horizontal line segments. There is an open circle at one end to indicate that a value of x may not have two function values.