Exponential Growth & Decay
Notes- Day 1 exploration

name Key
date _____ period 6

Exponential Functions: any function in the form $f(x) = a^x$, where a is a constant

$y = ab^x \quad b \neq 0$

a = initial amount

b = growth/decay factor

x = time interval

*When $b > 1$ the equation is a growth function
*When $0 < b < 1$ the equation is a decay function

b = 1 ± r (where r is your growth/decay rate) usually given as % ⇒ must convert to decimal

To find the "b" **convert** the growth/decay **rate** from a percent **to a decimal**, then...

→ **(b = 1 + r)** Exponential **GROWTH**: Add the decimal to 1

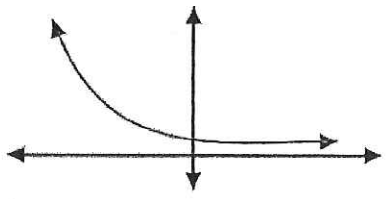
→ **(b = 1 – r)** Exponential **DECAY**: Subtract the decimal from 1

If it says the data is doubling, use 2 for b.

If it says the data is tripling, use 3 for b.

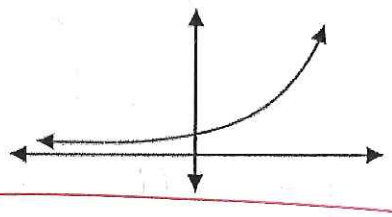
Graph of Exponential decay

C.



Graph of Exponential growth

D.



Data in table of an exponential relationship
- interval changes by a constant factor

x	y	
1	2	×2
2	4	×2
3	8	×2
4	16	

$y = a b^x$ (b)

1. A rabbit population triples every year. If I start with 4 rabbits, how many rabbits will I have in 7 years?

growth or decay? Growth

growth/decay rate (r)= N/A growth/decay factor (b)= 3

initial amount (a) = 4 time interval (x) = 7

$y = 4 \cdot 3^7$

$y =$ 8,748 rabbits

In calc $y = 4(3)^x$ 2nd table find $x = 7$

2. The new drug to treat SARS has a half-life of 3 hours. If 50cc is initially administered, how much will still be in your system 24 hours later?

growth or decay? decay

growth/decay rate (r)= n/A growth/decay factor (b)= $\frac{1}{2}$ or 0.5

initial amount (a) = 50 time interval (x) = $\frac{24}{3} = 8$

$y = 50\left(\frac{1}{2}\right)^8$

$y = 0.195$ $\approx$ 0.20 cc's

3. Evans town's population in the year 1910 was 4,210. If we know that the population grew at a rate of 4.5% per year, what was the population in the year 1936?

growth or decay? Growth

growth/decay rate (r)= 4.5% .045 growth/decay factor (b)= 1 + .045 = 1.045

initial amount (a) = 4210 time intervals (x) = 36 - 10 = 26

$y = 4210(1.045)^{26}$

$y = 13222.2$

$\approx$ 13,222 people

4. $500 is invested in a bank for 18 years. If the money made 12.5% each year, what was the value when the account was closed?

growth or decay? Growth

growth/decay rate (r)= 12.5% → 0.125 growth/decay factor (b)= 1 + 0.125 = 1.125

initial amount (a) = 500 time intervals (x) = 18

$y = 500(1.125)^{18}$

$y = 4165.962\ldots$ $4,165.92

5. In 1998, the population of humpback whales began decreasing (decay) at a rate of 7.4% per year. If there were 176,534 whales in 1998, how many should they expect in 2015?

$a = 176{,}534$ rate: $7.4\% \to .074$ $b = 1 - 0.074 = 0.926$

$x = 17$

$y = 176{,}534(0.926)^{17}$

$y = 47776.5$

$\approx 47{,}777$ whales

6. Ms. Hirsch just bought a Honda Accord worth $25,000. The car's value will depreciate (decay) by about 5.5% every year. How much will the car be worth in 6 years when the loan is paid off?

$a = 25{,}000$ rate: $5.5\% \to 0.055$ $b = 1 - 0.055 = 0.945$

$x = 6$

$y = 25{,}000(0.945)^{6}$

$y = 17804.54$

$\$17{,}804.54$

7. The population of white sea otters is declining at a rate of 2.5% each year. If the current population is 1.9 million, how many white sea otters were there 5 years ago? (past)

$a = 1{,}900{,}000$ rate: $2.5\% \to 0.025$ $b = 1 - 0.025 = 0.975$ $x = -5$

$y = 1{,}900{,}000(0.975)^{-5}$

$y = 2156405.9$

$\approx 2{,}156{,}406$ sea otters

8. Suppose you have 30,000 bacteria in a petri dish. You know that the culture doubles (growth) every hour. How many bacteria did you have 5 hours ago?

$a = 30{,}000$

$x = -5$

$b = 2$

$y = 30{,}000(2)^{-5}$

$y = 937.5$

≈ 938 bacteria

9. Ten years ago, the Smiths bought a house for $96,000. Their home is now worth $125,000. Assuming there has been a steady growth rate, what was the annual rate of appreciation?

$a = 96000$

$y = 125000$

$b = ?$ (1+r)

$x = 10$

$\frac{125{,}000}{96000} = \frac{96{,}000\,b^{10}}{96000}$

$\sqrt[10]{1.302} = \sqrt[10]{b^{10}}$

$1.0267 = b$

on calc: 10 math 5: $\sqrt[x]{}$ 1.032 enter

$r + 1 = b$

$r + 1 = 1.0267$

$-1 \quad -1$

$r = 0.0267$

$r = \frac{0.0267}{100} =$

$\approx 2.67\%$

10. Mr. Adams bought his car for $28,000 five years ago. If it is now worth $18,000, what was the annual rate of depreciation? $b = 1 - r$

$a = 28000$

$y = 18000$

$b = ?$ $(1-r)$

$x = 5$

$\frac{18000}{28000} = \frac{28000\,b^{5}}{28000}$

$\sqrt[5]{.643} = \sqrt[5]{b^{5}}$

$.915 = b$

$.915 = 1 - r$

$-1 \quad -1$

$-.0845 = -r$

$-1 \quad -1$

$r = 0.0845$

$r = \frac{0.0845}{100}$

$\approx 8.45\%$