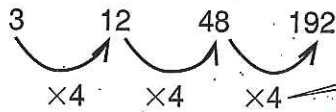


LESSON

11-1 Geometric Sequences

In a **geometric sequence**, each term is *multiplied* by the same number to get to the next term. This number is called the **common ratio**.



The common ratio is 4.

Yes

Determine if the sequence 2, 6, 18, 54, ... is a geometric sequence.

Divide each term by the term before it.

$\frac{54}{18} = 3$ $\frac{18}{6} = 3$ $\frac{6}{2} = 3$

This is a geometric sequence; 3 is the common ratio.

NO

Determine if the sequence 5, 10, 15, 20, ... is a geometric sequence.

Divide each term by the term before it.

$\frac{20}{15} = \frac{4}{3}$ $\frac{15}{10} = \frac{3}{2}$ $\frac{10}{5} = 2$

This is not a geometric sequence; there is no common ratio.

Find the next three terms in the geometric sequence 1, 4, 16, 64,

Step 1: Find the common ratio.

$\frac{64}{16} = 4$ $\frac{16}{4} = 4$ $\frac{4}{1} = 4$

Step 2: Continue to multiply by the common ratio.

$64 \times 4 = 256$ $256 \times 4 = 1024$ $1024 \times 4 = 4096$

The next three terms are 256, 1024, and 4096.

Determine if each sequence is a geometric sequence. Explain.

- 2, 4, 6, 8, ... $\frac{4}{2} = 2$ $\frac{6}{4} = 1.5$ NO, no common ratio
- 4, 8, -16, 32, ... $\frac{8}{-4} = -2$ $\frac{-16}{8} = -2$ Yes, the common ratio is -2
- 32, 16, 8, 4, ... Yes, the common ratio is 0.5 ($\frac{1}{2}$)

Find the common ratio in each geometric sequence below. Then find the next three terms.

4. 1, 5, 25, 125, ... $r = 5$

625, 3125, 15,625

5. -6, 12, -24, 48, ... $r = \frac{12}{-6} = -2$

-96, 192, -384

6. 4, 6, 9, 13.5, ... $r = \frac{6}{4} = 1.5$

20.25, 30.375, 45.5625

7. $\frac{1}{4}, \frac{1}{2}, 1, 2, \dots$ $r = \frac{2}{1} = 2$

4, 8, 16

LESSON

11-1 Geometric Sequences (continued)

There are two ways to find a given term of a geometric sequence.

Find the 8th term in the geometric sequence 5, 10, 20, 40,

Method 1: Extend the sequence to the 8th term.

Step 1: Find the common ratio.

$$\frac{40}{20} = 2 \quad \frac{20}{10} = 2 \quad \frac{10}{5} = 2$$

The common ratio is 2.

Step 2: Continue to multiply each term by 2.

5, 10, 20, 40, 80, 160, 320, **640**, ...

Stop at the 8th term.

The 8th term is 640.

Method 2: Use a formula to find the 8th term.

Look at Method 1. The first term, 5, was multiplied by 2 seven times to get to the eighth term.

$$\text{8th term} = 5(2)(2)(2)(2)(2)(2)(2) = 5(2)^7$$

Written as a formula, this would be:

$$a_n = a_1 r^{n-1}$$

where n is the number of terms and r is the common ratio.

To find the 8th term of the sequence, use $n = 8$ and $r = 2$.

$$a_n = a_1 r^{n-1}$$

$$a_8 = 5(2)^{8-1}$$

$$a_8 = 5(2)^7$$

$$a_8 = 5(128)$$

$$a_8 = 640$$

The 8th term is 640.

Find the indicated term.

$$a_n = a_1 (r)^{n-1}$$

8. $a_1 = 7, r = -2$; 10th term

$$a_{10} = 7(-2)^{10-1}$$

$$a_{10} = -3584$$

9. $a_1 = -4, r = 3$; 8th term

$$a_8 = -4(3)^{8-1}$$

$$a_8 = -8748$$

10. The first term of a geometric sequence is 2, and the common ratio is 3. What is the 7th term?

$$a_7 = 2(3)^{7-1} \quad a_7 = 458$$

11. The first term of a geometric sequence is -3, and the common ratio is -2. What is the 9th term?

$$a_9 = -3(-2)^{9-1} \quad a_9 = -768$$

12. Find the 12th term in the geometric sequence 5, -15, 45, -135,

$$a_{12} = 5(-3)^{12-1} \quad a_{12} = -885,735$$

13. Find the 8th term in the geometric sequence 243, 81, 27, 9,

$$a_8 = 243\left(\frac{1}{3}\right)^{8-1} \quad a_8 = \frac{1}{9}$$

$$r = \frac{-15}{5} = -3$$

$$r = \frac{81}{243} = \frac{1}{3}$$