

10 problems from each side

10-2 Study Guide and Intervention

Operations with Radical Expressions

Add and Subtract Radical Expressions When adding or subtracting radical expressions, use the Associative and Distributive Properties to simplify the expressions. If radical expressions are not in simplest form, simplify them.

Example 1 Simplify $10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6}$.

$$10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6} = (10 - 4)\sqrt{6} + (-5 + 6)\sqrt{3} \quad \text{Associative and Distributive Properties}$$

$$= 6\sqrt{6} + \sqrt{3} \quad \text{Simplify.}$$

Example 2 Simplify $3\sqrt{12} + 5\sqrt{75}$.

$$\begin{aligned} 3\sqrt{12} + 5\sqrt{75} &= 3\sqrt{2^2 \cdot 3} + 5\sqrt{5^2 \cdot 3} && \text{Simplify.} \\ &= 3 \cdot 2\sqrt{3} + 5 \cdot 5\sqrt{3} && \text{Simplify.} \\ &= 6\sqrt{3} + 25\sqrt{3} && \text{Simplify.} \\ &= 31\sqrt{3} && \text{Distributive Property} \end{aligned}$$

Exercises

Simplify.

1. $2\sqrt{5} + 4\sqrt{5}$

$$\boxed{6\sqrt{5}}$$

2. $\sqrt{6} - 4\sqrt{6}$

$$\boxed{-3\sqrt{6}}$$

3. $\sqrt{8} - \sqrt{2}$

$$\boxed{\sqrt{2}}$$

4. $3\sqrt{75} + 2\sqrt{5}$

$$\boxed{15\sqrt{3} + 2\sqrt{5}}$$

5. $\sqrt{20} + 2\sqrt{5} - 3\sqrt{5}$

$$\boxed{\sqrt{5}}$$

6. $2\sqrt{3} + \sqrt{6} - 5\sqrt{3}$

$$\boxed{-3\sqrt{3} + \sqrt{6}}$$

7. $\sqrt{12} + 2\sqrt{3} - 5\sqrt{3}$

$$\boxed{-\sqrt{3}}$$

8. $3\sqrt{6} + 3\sqrt{2} - \sqrt{50} + \sqrt{24}$

$$\boxed{-2\sqrt{2} + 5\sqrt{6}}$$

9. $\sqrt{8a} - \sqrt{2a} + 5\sqrt{2a}$

$$\boxed{6\sqrt{2a}}$$

10. $\sqrt{54} + \sqrt{24}$

$$\boxed{5\sqrt{6}}$$

11. $\sqrt{3} + \sqrt{\frac{1}{3}}$

$$\boxed{\frac{4\sqrt{3}}{3}}$$

12. $\sqrt{12} + \sqrt{\frac{1}{3}}$

$$\boxed{\frac{7\sqrt{3}}{3}}$$

13. $\sqrt{54} - \sqrt{\frac{1}{6}}$

$$\boxed{\frac{17\sqrt{6}}{6}}$$

14. $\sqrt{80} - \sqrt{20} + \sqrt{180}$

$$\boxed{8\sqrt{5}}$$

15. $\sqrt{50} + \sqrt{18} - \sqrt{75} + \sqrt{27}$

$$\boxed{18\sqrt{2} - 2\sqrt{3}}$$

$$16. 2\sqrt{3} - 4\sqrt{45} + 2\sqrt{\frac{1}{3}}$$

$$\boxed{\frac{8\sqrt{3}}{3} - 12\sqrt{5}}$$

17. $\sqrt{125} - 2\sqrt{\frac{1}{5}} + \sqrt{\frac{1}{3}}$

$$\boxed{\frac{23\sqrt{5}}{5} + \frac{\sqrt{3}}{3}}$$

$$18. \sqrt{\frac{2}{3}} + 3\sqrt{3} - 4\sqrt{\frac{1}{12}}$$

$$\boxed{\frac{7\sqrt{3}}{3} + \frac{\sqrt{6}}{3}}$$

10-2 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Multiply Radical Expressions Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example Multiply $(3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8})$.

Use the FOIL method.

$$\begin{aligned}
 (3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8}) &= (3\sqrt{2})(4\sqrt{20}) + (3\sqrt{2})(\sqrt{8}) + (-2\sqrt{5})(4\sqrt{20}) + (-2\sqrt{5})(\sqrt{8}) \\
 &= 12\sqrt{40} + 3\sqrt{16} - 8\sqrt{100} - 2\sqrt{40} && \text{Multiply.} \\
 &= 12\sqrt{2^2 \cdot 10} + 3 \cdot 4 - 8 \cdot 10 - 2\sqrt{2^2 \cdot 10} && \text{Simplify.} \\
 &= 24\sqrt{10} + 12 - 80 - 4\sqrt{10} && \text{Simplify.} \\
 &= 20\sqrt{10} - 68 && \text{Combine like terms.}
 \end{aligned}$$

Exercises Find each product.

1. $2(\sqrt{3} + 4\sqrt{5})$ $2\sqrt{3} + 8\sqrt{5}$

2. $\sqrt{6}(\sqrt{3} - 2\sqrt{6})$ $-12 + 3\sqrt{2}$

3. $\sqrt{5}(\sqrt{5} - \sqrt{2})$ $5 - \sqrt{10}$

4. $\sqrt{2}(3\sqrt{7} + 2\sqrt{5})$ $2\sqrt{14} + 3\sqrt{10}$

5. $(2 - 4\sqrt{2})(2 + 4\sqrt{2})$ -28

6. $(3 + \sqrt{6})^2$ $15 + 6\sqrt{6}$

7. $(2 - 2\sqrt{5})^2$ $24 - 8\sqrt{5}$

8. $3\sqrt{2}(\sqrt{8} + \sqrt{24})$ $12 + 12\sqrt{3}$

9. $\sqrt{8}(\sqrt{2} + 5\sqrt{8})$ 44

10. $(\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 3\sqrt{2})$ -13

11. $(\sqrt{3} + \sqrt{6})^2$ $9 + 6\sqrt{2}$

12. $(\sqrt{2} - 2\sqrt{3})^2$ $14 - 4\sqrt{6}$

13. $(\sqrt{5} - \sqrt{2})(\sqrt{2} + \sqrt{6})$
 $-2 - 2\sqrt{3} + \sqrt{10} + \sqrt{30}$

14. $(\sqrt{8} - \sqrt{2})(\sqrt{3} + \sqrt{6})$
 $2\sqrt{3} + \sqrt{6}$

15. $(\sqrt{5} - \sqrt{18})(7\sqrt{5} + \sqrt{3})$
 $35 - 3\sqrt{15} - 2\sqrt{18} + \sqrt{15}$

16. $(2\sqrt{3} - \sqrt{45})(\sqrt{12} + 2\sqrt{6})$
 $12 + 12\sqrt{2} - 6\sqrt{15} - 6\sqrt{30}$

17. $(2\sqrt{5} - 2\sqrt{3})(\sqrt{10} + \sqrt{6})$
 $14\sqrt{2}$

18. $(\sqrt{2} + 3\sqrt{3})(\sqrt{12} - 4\sqrt{8})$
 $2 - 22\sqrt{6}$

10-2 Add/Subtract Radicals

Practice Key

$$3. \sqrt{8} - \sqrt{2}$$

$$2\sqrt{2} - \sqrt{2} = \boxed{\sqrt{2}}$$

$$4. 3\sqrt{75} + 2\sqrt{5}$$

$$\begin{array}{r} 3 \\ \times 25 \\ \hline 75 \end{array}$$

$$\boxed{15\sqrt{3} + 2\sqrt{5}}$$

$$5. \sqrt{20} + 2\sqrt{5} - 3\sqrt{5}$$

$$\begin{array}{r} 4 \\ \times 5 \\ \hline 20 \end{array}$$

$$2\sqrt{5} + 2\sqrt{5} - 3\sqrt{5} = \boxed{\sqrt{5}}$$

$$6. \sqrt{12} + 2\sqrt{3} - 5\sqrt{3}$$

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 12 \end{array}$$

$$2\sqrt{3} + 2\sqrt{3} - 5\sqrt{3} = \boxed{-\sqrt{3}}$$

$$8. 3\sqrt{6} + 3\sqrt{2} - \sqrt{50} + \sqrt{24}$$

$$\begin{array}{r} 2 \ 25 \\ \times 55 \\ \hline 55 \end{array} \quad \begin{array}{r} 4 \ 6 \\ \times 22 \\ \hline 22 \end{array}$$

$$(3\sqrt{6}) + (3\sqrt{2}) - \boxed{5\sqrt{2}} + (2\sqrt{6}) = \boxed{-2\sqrt{2} + 5\sqrt{6}}$$

$$9. \sqrt{8a} - \sqrt{2a} + 5\sqrt{2a}$$

$$\begin{array}{r} 2 \ 4 \\ \times 2 \\ \hline 4 \end{array}$$

$$2\sqrt{2a} - \sqrt{2a} + 5\sqrt{2a} = \boxed{6\sqrt{2a}}$$

$$10. \sqrt{54} + \sqrt{24}$$

$$\begin{array}{r} 2 \ 27 \\ \times 33 \\ \hline 27 \end{array} \quad \begin{array}{r} 2 \ 12 \\ \times 24 \\ \hline 12 \end{array}$$

$$3\sqrt{6} + 2\sqrt{6} = \boxed{5\sqrt{6}}$$

$$11. \sqrt{3} + \sqrt{\frac{1}{3}}$$

$$\sqrt{3} + \frac{1}{\sqrt{3}} \quad \sqrt{3} + \frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$$

$$\sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$$

$$12. \sqrt{12} + \sqrt{\frac{1}{3}}$$

$$2\sqrt{3} + \sqrt{3} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$$

$$2\sqrt{3} + \frac{\sqrt{3}}{3} = \boxed{\frac{7\sqrt{3}}{3}}$$

$$13. \sqrt{54} - \sqrt{\frac{1}{6}}$$

$$3\sqrt{6} - \frac{1}{\sqrt{6}} \left(\frac{\sqrt{6}}{\sqrt{6}} \right)$$

$$3\sqrt{6} - \frac{\sqrt{6}}{6} = \frac{17\sqrt{6}}{6}$$

$$14. \sqrt{80} - \sqrt{20} + \sqrt{180}$$

$\begin{array}{c} 2 \\ \cancel{2} \end{array} \begin{array}{c} 4 \\ \cancel{4} \\ 0 \end{array}$
 $\begin{array}{c} 2 \\ \cancel{2} \end{array} \begin{array}{c} 1 \\ \cancel{1} \\ 0 \end{array}$
 $\begin{array}{c} 2 \\ \cancel{2} \end{array} \begin{array}{c} 9 \\ \cancel{4} \\ 5 \end{array}$
 $\begin{array}{c} 2 \\ \cancel{2} \end{array} \begin{array}{c} 1 \\ \cancel{1} \\ 0 \end{array}$
 $\begin{array}{c} 3 \\ \cancel{3} \end{array} \begin{array}{c} 1 \\ \cancel{1} \\ 5 \end{array}$
 $\begin{array}{c} 3 \\ \cancel{3} \end{array} \begin{array}{c} 1 \\ \cancel{1} \\ 5 \end{array}$

$$15. \sqrt{50} + \sqrt{18} - \sqrt{75} + \sqrt{27}$$

$\begin{array}{c} 2 \\ \cancel{2} \\ 5 \end{array}$
 $\begin{array}{c} 2 \\ \cancel{2} \\ 3 \end{array}$
 $\begin{array}{c} 3 \\ \cancel{3} \\ 5 \end{array}$
 $\begin{array}{c} 3 \\ \cancel{3} \\ 7 \end{array}$

$$4\sqrt{5} - 2\sqrt{5} + 6\sqrt{5} = \boxed{8\sqrt{5}}$$

$$\begin{array}{r} 5\sqrt{2} + 3\sqrt{2} - 5\sqrt{3} + 3\sqrt{3} \\ \hline 18\sqrt{2} - 2\sqrt{3} \end{array}$$

$$16. 2\sqrt{3} - 4\sqrt{45} + 2\sqrt{\frac{1}{3}}$$

$\begin{array}{c} 3 \\ \cancel{3} \\ 5 \end{array}$
 $2\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}$

$$2\sqrt{3} - 12\sqrt{5} + \frac{2\sqrt{3}}{3} = \boxed{\frac{8\sqrt{3}}{3} - 12\sqrt{5}}$$

$$17. \sqrt{125} - 2\sqrt{5} + \sqrt{\frac{1}{3}}$$

$\begin{array}{c} 5 \\ \cancel{5} \\ 5 \end{array}$
 $-2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{-2\sqrt{5}}{5}$

$$5\sqrt{5} - \frac{2\sqrt{5}}{5} + \frac{\sqrt{3}}{3} = \boxed{\frac{23\sqrt{5}}{5} + \frac{\sqrt{3}}{3}}$$

$\frac{25\sqrt{5}}{5} - \frac{2\sqrt{5}}{5}$

$$18. \sqrt{\frac{2}{3}} + 3\sqrt{3} - 4\sqrt{\frac{1}{12}}$$

$$\frac{\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{6}}{3} \quad -\frac{4\sqrt{1}}{\sqrt{12}} = \frac{-4}{\cancel{\sqrt{12}}} \cancel{\frac{8-4}{2\sqrt{3}}} = \frac{-2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-2\sqrt{3}}{3}$$

$$\frac{\sqrt{6}}{3} + 3\sqrt{3} - \frac{2\sqrt{3}}{3}$$

$\frac{9\sqrt{3}}{3} - \frac{2\sqrt{3}}{3}$

$$\frac{7\sqrt{3}}{3} + \frac{\sqrt{6}}{3}$$

10-2 Multiply Radicals Practice Key

$$1. 2(\sqrt{3} + 4\sqrt{5}) = \boxed{2\sqrt{3} + 8\sqrt{5}}$$

$$2. \sqrt{6}(\sqrt{3} - 2\sqrt{6}) = \sqrt{18} - 2\sqrt{36} = \boxed{3\sqrt{2} - 12}$$

$\downarrow^{2\sqrt{3}} \quad \downarrow^{2(6)}$

$$\boxed{-12 + 3\sqrt{2}}$$

$$3. \sqrt{5}(\sqrt{5} - \sqrt{2}) = \sqrt{5} - \sqrt{10} = \boxed{5 - \sqrt{10}}$$

$$4. \sqrt{2}(3\sqrt{7} + 2\sqrt{5}) = \boxed{3\sqrt{14} + 2\sqrt{10}}$$

$$5. (2-4\sqrt{2})(2+4\sqrt{2}) = 4 - 8\sqrt{2} - 8\sqrt{2} - 32 = \boxed{-28}$$

$$6. (3+\sqrt{6})^2 = (3+\sqrt{6})(3+\sqrt{6})$$

$$\boxed{15 + 6\sqrt{6}}$$

3	$\sqrt{6}$	$\sqrt{1}$
9	$3\sqrt{6}$	3
$3\sqrt{6}$	6	$\sqrt{1}$

CLT

$$7. (2-2\sqrt{5})^2 = (2-2\sqrt{5})(2-2\sqrt{5})$$

$$= 4 - 4\sqrt{5} - 4\sqrt{5} + 4\sqrt{25} = \boxed{24 - 8\sqrt{5}}$$

$\downarrow^{2\sqrt{5}}$

$$8. 3\sqrt{2}(\sqrt{8} + \sqrt{24}) = 3\sqrt{16} + 3\sqrt{48} = \boxed{12 + 12\sqrt{2}}$$

$\downarrow^{4\sqrt{2}}$ $\downarrow^{3\sqrt{24}}$

$$9. \sqrt{8}(\sqrt{2} + 5\sqrt{8}) = \sqrt{16} + 5\sqrt{64} = \boxed{144}$$

$\downarrow^{4\sqrt{2}}$ $\downarrow^{8\sqrt{8}}$

$$10. (\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 3\sqrt{2}) = F - I - L$$

$$= 5 + 3\sqrt{10} - 3\sqrt{10} - 18$$

$$\boxed{-13}$$

$$11. (\sqrt{3} + \sqrt{6})^2 = (\sqrt{3} + \sqrt{6})(\sqrt{3} + \sqrt{6}) = F O I L$$

$$= 3 + \sqrt{18} + \sqrt{18} + 6$$

$$= 9 + 2\sqrt{18}$$

$\begin{matrix} 2 \\ 9 \\ \hline 3 \\ 3 \end{matrix}$

$$= \boxed{9 + 6\sqrt{2}}$$

$$12. (\sqrt{2} - 2\sqrt{3})^2 = (\sqrt{2} - 2\sqrt{3})(\sqrt{2} - 2\sqrt{3}) = F O I L$$

$$= 2 - 2\sqrt{6} - 2\sqrt{6} + 4\sqrt{9}$$

$\begin{matrix} 2 \\ 2\sqrt{6} \\ 2\sqrt{6} \\ 4\sqrt{9} \\ \hline 12 \end{matrix}$

$$= \boxed{14 - 4\sqrt{6}}$$

$$13. (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{6}) = F O I L$$

$$= \sqrt{10} + \sqrt{30} - \sqrt{4} - \sqrt{12}$$

$\begin{matrix} 2\sqrt{5} \\ 3\sqrt{5} \\ \hline 2\sqrt{2} \\ 2\sqrt{2} \\ 2\sqrt{3} \end{matrix}$

$$= \sqrt{10} + \sqrt{30} - 2 - 2\sqrt{3}$$

$$14. (\sqrt{8} - \sqrt{2})(\sqrt{3} + \sqrt{6}) = F O I L$$

$$= \sqrt{24} + \sqrt{48} - \sqrt{6} - \sqrt{12}$$

$\begin{matrix} 2\sqrt{2} \\ 2\sqrt{2} \\ 2\sqrt{3} \\ \hline 2\sqrt{2} \\ 2\sqrt{2} \\ 2\sqrt{3} \end{matrix}$

$$= 2\sqrt{6} + 4\sqrt{3} - \sqrt{6} - 2\sqrt{3}$$

$$= \boxed{2\sqrt{3} + \sqrt{6}}$$

$$15. (\sqrt{5} - \sqrt{18})(\sqrt{5} + \sqrt{3}) = F O I L$$

$$= \sqrt{25} - \sqrt{18}$$

25	-18
25	-210
35	-210
$\sqrt{15}$	$\sqrt{64}$
	-3\sqrt{4}

$$= \boxed{35 - 3\sqrt{6} - 21\sqrt{10} + \sqrt{15}}$$

$$16. (2\sqrt{3} - \sqrt{45})(\sqrt{2} + 2\sqrt{6}) \xrightarrow{\text{Simplify!}} (2\sqrt{3} - 3\sqrt{5})(2\sqrt{3} + 2\sqrt{6})$$

F O I L

$$= 4\sqrt{9} + 4\sqrt{18} - 6\sqrt{15} - 6\sqrt{3}$$

$\begin{matrix} 0 \\ 5 \\ 5 \end{matrix}$

$$= \boxed{12 + 12\sqrt{2} - 6\sqrt{15} - 6\sqrt{3}}$$