Name: Key Class Period ________

Quarter 3 QCA Review

Unit 5: Radical Functions

1. Simplify completely.

a.) $(a-4)^2$

$(a-4)(a-4)$

$a^2-4a-4a+16$

$\boxed{a^2-8a+16}$

b.) $(2a^2+4a-7)+(3a^2-a-2)$ CLT

$\boxed{5a^2+3a-9}$

c.) $2(5x+2)-2(4x-3)$

$10x+4-8x+6$

$\boxed{2x+10}$

d.) $(-9x^2y)\left(\frac{1}{3}xy\right)^2$

$(-9x^2y)(\frac{1}{9}x^2y^2)$

$\boxed{(-x^4y^3)}$

e.) $(4a^2b)(-5a^3b^2)(6a^4)$

$\boxed{-120a^9b^3}$

f.) $(-2a^2b^2)^3(-3ab^4)$

$(-8a^6b^6)(-3ab^4)$

$\boxed{24a^7b^{10}}$

g.) $-2x(3x^2-3x+2)$

$\boxed{-6x^3+6x^2-4x}$

h.) $(3x+2)(x+3)$

$3x^2+9x+2x+6$

$\boxed{3x^2+11x+6}$

i.) $(-2a^2-5a-7)-(-3a^2+7a+1)$

$-2a^2-5a-7+3a^2-7a-1$

$\boxed{a^2-12a-8}$

2. Simplify each expression

a. $\sqrt{75}=\boxed{5\sqrt{3}}$ (3, 25; 5, 5)

b. $\sqrt{12}=\boxed{2\sqrt{3}}$ (4, 3; 2, 2)

c. $\sqrt[3]{54}=\boxed{3\sqrt[3]{2}}$ (6, 9; 2, 3, 3, 3)

d. $3\sqrt{50}=\boxed{15\sqrt{2}}$ (2, 25; 5, 5)

e. $5\sqrt{27}=\boxed{15\sqrt{3}}$ (3, 9; 3, 3)

f. $\sqrt{36x^6y^2}$ $\boxed{6x^3y}$ (6, 6; 2, 3, 2, 3)

g. $\sqrt[3]{81a^3b^2c}$ $\boxed{3a\sqrt[3]{3b^2c}}$ (9, 9; 3, 3, 3, 3)

h. $\sqrt{50xy^2z^5}=\boxed{5yz^2\sqrt{2xz}}$ (25, 2; 5, 5)

j. $3\sqrt{5}+6\sqrt{5}$ $\boxed{9\sqrt{5}}$

3. Write using rational exponents:

a. $\sqrt[3]{x^5}=\boxed{x^{\frac{5}{3}}}$

b. $\sqrt{x^3}=\boxed{x^{\frac{3}{2}}}$

c. $\sqrt[5]{6^3}=\boxed{6^{\frac{3}{5}}}$

4. Write in radical form:

a. $2x^{\frac{2}{3}}$ = $\boxed{2\sqrt[3]{x^2}}$

b. $(3x)^{\frac{1}{5}}$ = $\sqrt[5]{3x}$

c. $25^{\frac{3}{2}}$ = $\boxed{\sqrt{25^3}}$

Transformations

5. Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation of the graph $f(x) = 3\sqrt{x+2} - 5$.

- vertical stretch
- left 2
- down 5

6. Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation of the graph $h(x) = -\sqrt{x} - 7$.

- reflected over the x-axis
- down 7

7. **(PREAP Only)** Using the graph $f(x) = \sqrt[3]{x}$ as a guide, describe the transformation of the graph $g(x) = -\sqrt[3]{2(x-1)} + 4$.

- reflected over the x-axis
- up 4
- horizontal compression
- right 1

8. The parent function $f(x) = \sqrt{x}$ is stretched vertically by a factor of 4, reflected across the *x*-axis, and translated left 2 units. Write the square-root function *g(x)*.

$g(x) = -4\sqrt{x+2}$

9. The function *g* is a translation 3 units left and 2 units up of from $f(x) = \sqrt{x} + 9$. Write the function *g*(*x*).

$g(x) = \sqrt{x+3} + 11$

Applications

10. The velocity V of an object in meters per second can be defined as $v=\sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy. Find the kinetic energy of an object with velocity of 8 meters per second and a mass of 5 grams

$\downarrow$ V $\quad$ $\downarrow$ m

$8=\sqrt{\frac{2K}{5}}$ $\quad 64=\frac{2K}{5}$ $\quad 320=2K$ $\quad \boxed{160=K}$

11. Mr. Ingram's physics class is experimenting with pendulums. The class learned the formula $T=2\pi\sqrt{\frac{L}{g}}$, which relates the time T in seconds that it takes for a pendulum to swing back and forth based on gravity g (32 feet per second squared) and the length of the pendulum L in feet. One group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?

$\frac{1.76 = 2\pi\sqrt{\frac{L}{32}}}{2\pi}$ $\quad .28=\sqrt{\frac{L}{32}}$ $\quad .078=\frac{L}{32}$

$\boxed{2.51 = L \text{ feet}}$

12. On an interstate highway under dry conditions, the maximum safe speed in miles per hour around a curve with radius of r in feet is approximated by the equation $f(r)=\sqrt{1.6r}$. What would be the maximum safe speed around a curve with a radius of 1000 feet?

$f(r)=\sqrt{1.6(1000)}$

$\boxed{f(r) = 40 \text{ feet}}$

Solve the following radical equations and then check your answer.

13. $\sqrt{3x+2}+4=6$

$\sqrt{3x+2}=2$

$3x+2=4$

$3x=2$

$\boxed{x=\frac{2}{3}}$ ✓

check

$\sqrt{3(\frac{2}{3})+2}+4=6$

$\sqrt{2+2}+4=6$

$\sqrt{4}+4=6$

$2+4=6$

$6=6$ ✓

14. $(\sqrt{5x-2})^2=(\sqrt{4x+8})^2$

$5x-2=4x+8$

$-4x \quad -4x$

$x-2=8$

$+2 \quad +2$

$\boxed{x=10}$ ✓

check

$\sqrt{5(10)-2}=\sqrt{4(10)+8}$

$\sqrt{50-2}=\sqrt{40+8}$

$\sqrt{48}=\sqrt{48}$ ✓

15. $4\sqrt{x+1}-10=-6$

$$4\sqrt{x+1}=4$$
$$\frac{4\sqrt{x+1}}{4}=\frac{4}{4}$$
$$\sqrt{x+1}=1$$
$$x+1=1$$
$$-1\quad -1$$
$$\boxed{x=0}\ \checkmark$$

$$4\sqrt{0+1}-10=-6$$
$$4\sqrt{1}-10=-6$$
$$4(1)-10=-6$$
$$4-10=-6$$
$$-6=-6\ \checkmark$$

16. $\left(5=\sqrt[3]{(8x+13)}\right)^3$

$$125=8x+13$$
$$-13\qquad -13$$
$$\frac{112=8x}{8}$$
$$\boxed{14=x}\ \checkmark$$

check

$$5=\sqrt[3]{8(14)+13}$$
$$5=\sqrt[3]{112+13}$$
$$5=\sqrt[3]{125}$$
$$5=5\ \checkmark$$

Find the inverse of the functions and state the domain & range of $f(x)$ and $f^{-1}(x)$:

17. $f(x)=x^2+7$ D: $\mathbb{R}$ R: $[7,\infty)$

$$x=y^2+7$$
$$x-7=y^2$$
$$\boxed{\pm\sqrt{x-7}=y}$$

Inverse D: $[7,\infty)$ R: $\mathbb{R}$

18. $f(x)=x^2-6$ D: $\mathbb{R}$ R: $[-6,\infty)$

$$x=y^2-6$$
$$x+6=y^2$$
$$\boxed{\pm\sqrt{x+6}=y}$$

Inverse D: $[-6,\infty)$ R: $\mathbb{R}$

19. $f(x)=\sqrt{x+4}-2$ D: $[-4,\infty)$ R: $[-2,\infty)$

$$x=\sqrt{y+4}-2$$
$$x+2=\sqrt{y+4}$$
$$(x+2)^2=y+4$$
$$\boxed{(x+2)^2-4=y}$$

needs domain restriction

Inverse D: $[-2,\infty)$ R: $[-4,\infty)$

20. $f(x)=\sqrt{x-3}+4$ D: $[3,\infty)$ R: $[4,\infty)$

$$x=\sqrt{y-3}+4$$
$$x-4=\sqrt{y-3}$$
$$(x-4)^2=y-3$$
$$\boxed{(x-4)^2+3=y}$$

needs domain restriction

Inverse D: $[4,\infty)$ R: $[3,\infty)$

21. Using the composition of functions, prove if $f(x)=\sqrt{x+4}-3$ and $g(x)=(x+3)^2-4$ are inverses of one another.

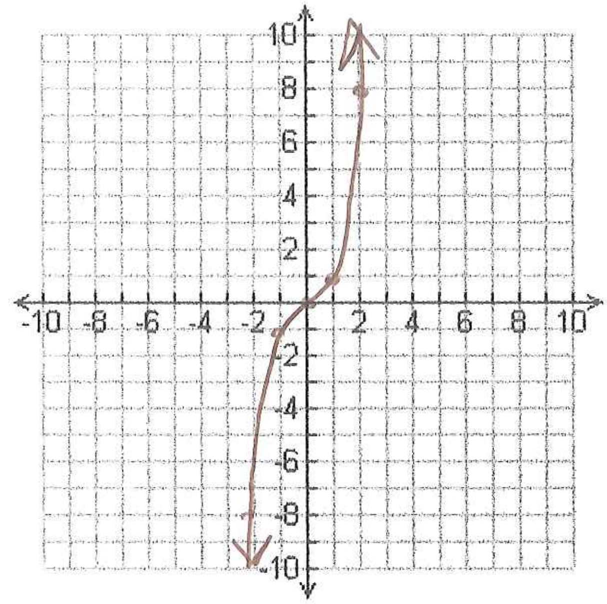
$$f(g(x))=\sqrt{(x+3)^2-4+4}-3$$
$$=\sqrt{(x+3)^2}-3$$
$$=x+3-3$$
$$=x\ \checkmark$$

they are inverses

22. Given the cubic parent function $f(x) = x^3$, complete the table, describe the domain, range and end behavior and graph.

x	y
-2	-8
-1	-1
Ø	1
1	1

2 8



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

End Behavior:

$x \to -\infty, f(x) \to$ $-\infty$

$x \to \infty, f(x) \to$ ∞

Factor Completely Sum & Differences of cubes

23. $27x^3 + 8$

$(3x+2)(9x^2 - 6x + 4)$

24. $a^3 - b^3$

$(a-b)(a^2 + ab + b^2)$

25. $64x^3 + 1$

$(4x+1)(16x^2 - 4x + 1)$

26. $8p^3 - 125q^3$

$(2p - 5q)(4p^2 + 10pq + 25q^2)$

Simplify

27. $(3x-2)(2x^2+3x-1)$

$6x^3 + 9x^2 - 3x - 4x^2 - 6x + 2$

$\boxed{6x^3 + 5x^2 - 9x + 2}$

28. $(x^3 + 3x^2 + 1)(3x^2 + 6x - 2)$

$3x^5 + 6x^4 - 2x^3 + 9x^4 + 18x^3 - 6x^2 + 3x^2 +$
$6x - 2$

$\boxed{3x^5 + 15x^4 + 16x^3 - 3x^2 + 6x - 2}$

Factor

29. $f(x) = x^3 + 9x^2 + 23x + 15$

	1	9	23	15
~~1~~	~~1~~	~~10~~	~~33~~	~~48~~
✓ -1	1	8	15	Ø

$(x+1)(x^2+8x+15)$

15
5 3
8

$\boxed{(x+1)(x+5)(x+3)}$

30. $f(x) = x^3 - 12x^2 + 47x - 60$

	1	-12	47	-60
~~1~~	~~1~~	~~-11~~	~~36~~	~~-24~~
✓ -1	1	-13	60	Ø

$\boxed{(x+1)(x^2-13x+60)}$

$\underbrace{\qquad\qquad}_{\text{non-factorable}}$

31. $f(x) = x^3 - x^2 - 14x + 24$

	1	-1	-14	24
~~1~~	~~1~~	~~0~~	~~-14~~	~~10~~
~~-1~~	~~1~~	~~-2~~	~~-12~~	~~36~~
✓ 2	1	1	-12	Ø

$(x-2)(x^2+x-12)$

-12
4 -3
1

$\boxed{(x-2)(x+4)(x-3)}$

32. $f(x) = x^4 + 3x^3 - 13x^2 - 15x$

$x\,\underbrace{(x^3+3x^2-13x-15)}_{\text{Factor}}$

	1	3	-13	-15
~~1~~	~~1~~	~~4~~	~~-9~~	~~-24~~
✓ -1	1	2	-15	Ø

$x(x+1)(x^2+2x-15)$

-15
5 -3
2

$\boxed{x(x+1)(x+5)(x-3)}$

Divide Using Any Method

33. $(x^3 - 22x^2 + 157x - 360) \div (x - 8)$

$$
\begin{array}{r|rrrr}
8 & 1 & -22 & 157 & -360 \\
 & \downarrow & 8 & -112 & 360 \\
\hline
 & 1 & -14 & 45 & 0
\end{array}
$$

$\boxed{x^2 - 14x + 45}$

34. $(3x^4 - 6x^3 - 6x^2 - 3x - 30) \div (x - 2)$

$$
\begin{array}{r|rrrrr}
2 & 3 & -6 & -6 & -3 & -30 \\
 & \downarrow & 6 & 0 & -12 & -30 \\
\hline
 & 3 & 0 & -6 & -15 & -60
\end{array}
$$

$\boxed{3x^3 - 6x - 15 - \frac{60}{x-2}}$

35. $(4x^3 - 12x^2 + 2x - 5) \div (x - 3)$

$$
\begin{array}{r}
4x^2 + 2 \\
x - 3 \overline{)\, 4x^3 - 12x^2 + 2x - 5} \\
-4x^3 + 12x^2 \qquad\qquad \\
\hline
2x - 5 \\
-2x + 6 \\
\hline
1
\end{array}
$$

$\boxed{4x^2 + 2 + \frac{1}{x-3}}$