

CLASS NOTES

Ratios and Proportions, Percent Change and Percents (Notes)

Name: KEY Date: _____ Period: _____

Example #1

A ratio is a comparison of two numbers by division. The ratio x to y can be expressed in the following ways: x to y $x:y$ $\frac{x}{y}$

An equation stating that two ratios are equal is called a proportion.

Do $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion? If you make $\frac{4}{5}$ 6 times bigger, it will be $\frac{24}{30}$.

$$\begin{array}{c} \times 6 \\ \curvearrowright \\ \frac{4}{5} = \frac{24}{30} \\ \curvearrowleft \\ \times 6 \end{array}$$

OR

cross products will be equal

$$\frac{4}{5} \times \frac{24}{30}$$

?

$$5 \times 24 = 30 \times 4$$

$$120 = 120 \text{ TRUE!}$$

Is each pair of ratios a proportion?

a) $\frac{6}{10} = \frac{2}{5}$
 ~~$30 \neq 20$~~

b) $\frac{1}{6} = \frac{5}{30}$ ✓
 $30 = 30$

c) $\frac{6}{8} = \frac{24}{28}$
 ~~$168 \neq 192$~~

Example #2

Solving proportions:

$$\frac{n}{15} = \frac{24}{16}$$

$$\frac{16n}{16} = \frac{360}{16}$$

$$n = 22.5$$

$$\frac{r}{8} = \frac{25}{40}$$

$$\frac{40r}{40} = \frac{200}{40}$$

$$r = 5$$

$$\frac{3.2}{4} = \frac{2.6}{n}$$

$$\frac{3.2n}{3.2} = \frac{10.4}{3.2}$$

$$n = 3.25$$

$$\frac{6}{14} = \frac{7}{x-3}$$

$$6(x-3) = 98$$

$$6x - 18 = 98$$

$$+18 \quad +18$$

$$\frac{6x}{6} = \frac{116}{6}$$

$$x = 19.3 \quad \boxed{\frac{58}{3}}$$

The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance d between these two places?

$$\frac{\text{inches}}{\text{miles}}$$

$$\frac{2}{9} = \frac{1.75}{d}$$

$$\frac{2d}{2} = \frac{15.75}{2}$$

$$\boxed{d = 7.875 \text{ miles}}$$

Example #3

Percent of change is a type of proportion. You can calculate the percent of change using the formula $\frac{\text{change}}{\text{original}} = \frac{x}{100}$. If it is marked up, it is a percent increase. If it is marked down, it is a percent decrease.

A shirt costs \$30 originally. It is marked down to \$25. What was the percent of change? $\approx 17\%$
 Was it an increase or decrease? decrease

$$\begin{array}{l} \text{Change} \rightarrow \frac{30-25}{30} = \frac{x}{100} \\ \text{Original} \rightarrow 30 \end{array}$$

$$\frac{5}{30} = \frac{x}{100}$$

$$\frac{30x}{30} = \frac{500}{30}$$

$$x = 16.6$$

exactly $\frac{50}{3}\%$
 $1 \text{ or } 16\frac{2}{3}\%$

$$\frac{\text{Change}}{\text{original}} = \frac{x\%}{100}$$

Original: \$25
New: \$28

change = 3

Increase or decrease?

$$\frac{3}{25} = \frac{x}{100}$$

$$\frac{25x}{25} = \frac{300}{25}$$

x = 12% increase

Original: 30
New: 12

Change = 30 - 12 = 18

Increase or decrease?

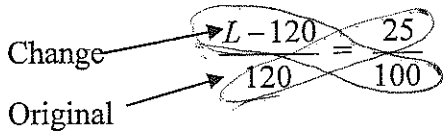
$$\frac{18}{30} = \frac{x}{100}$$

$$\frac{30x}{30} = \frac{1800}{30}$$

x = 60% decrease

The National Football League's (NFL) fields are 120 yards long. The Canadian Football League's (CFL) fields are 25% longer. How long is a CFL field?

Let L = the length of a CFL field. Since 25% is a percent of increase, an NFL field is shorter than a CFL field. Therefore, L - 120 represents the change.



$$100(L-120) = 2500$$

$$100L - 12000 = 2500$$

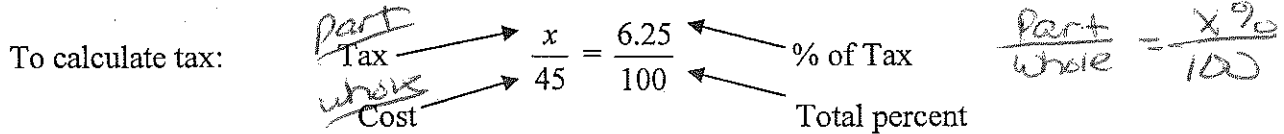
$$+12000 \quad +12000$$

$$\frac{100L}{100} = \frac{14500}{100}$$

$$L = 145 \text{ yards}$$

Example #4

A concert ticket costs \$45. If the sales tax is 6.25%, what is the total price of the ticket?



Tax on \$45 is $\frac{2.81}{100}$. $\$2.81 + \$45 = \$47.81$

Cost with Tax included

$$x = 2.8125$$

A new DVD costs \$24.99. If the sales tax is 7.25%, what is the total cost?

To calculate total including tax

Add % of tax + 100% item cost

$$\frac{\text{total cost}}{\text{cost}} = \frac{\% + 100}{100}$$

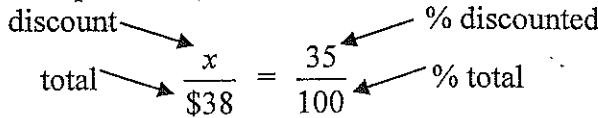
$$\frac{x}{24.99} = \frac{107.25}{100}$$

$$\frac{100x}{100} = \frac{2680.175}{100}$$

$$x = \$26.80$$

Example #5

A sweater is on sale for 35% off the original price. If the original price of the sweater is \$38, what is the discounted price?



$$\frac{100x}{100} = \frac{1330}{100}$$

$$x = 13.30$$

The final cost of the sweater will be \$38 - \$13.30 = \$24.70

total price discount Discount (\$ off) Discounted price

Equation: $T = X - 0.35x$

A picture frame originally priced at \$14.89 is on sale for 40% off. A tax of 8.25% is added on. What is the final cost?

Step 1: Find discounted price. $\frac{x}{14.89} = \frac{40}{100}$ $\frac{100x}{100} = \frac{595.6}{100}$ $x = 5.956$ (\$5.96)

Step 2: Calculate tax on discounted price.

Step 3: Add discounted price to tax for total cost owed.

$$\frac{x}{8.93} = \frac{108.25}{100}$$

$$x = 9.46$$

$$\frac{100x}{100} = \frac{966.6725}{100}$$

$$x = 9.6667$$

together