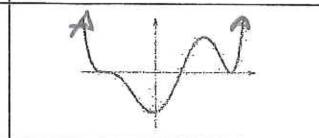
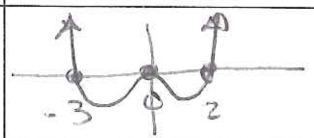
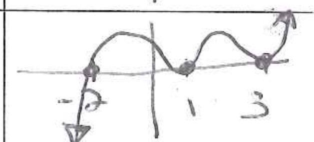
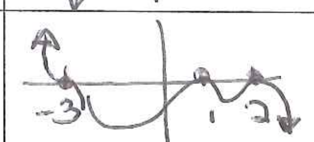
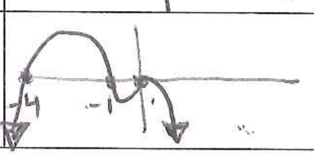
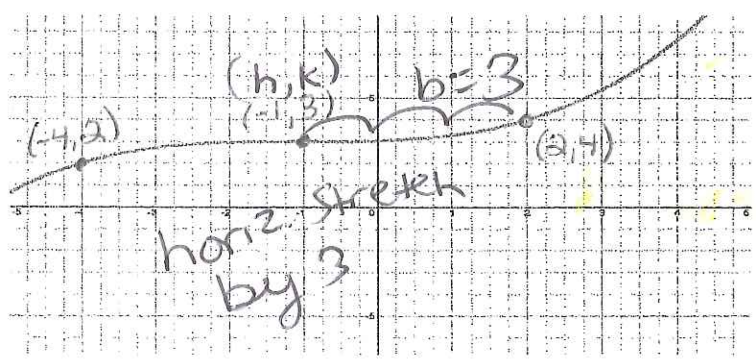
Key

Unit 4: Polynomial Functions Review – Pre-AP

Name: ______________________ Date: ______________ Period: _______

Function	Sketch graph of $f(x)$	List Roots / Describe action at the root			End Behavior $x \to -\infty, f(x) \to$	$x \to \infty, f(x) \to$
$ax^n = x^6$ ↑↑ $f(x)=(x+2)^3(x-1)^1(x-3)^2$		-2 cubic	1 linear	3 quadratic	$+\infty$	∞
$ax^n = 2x^4$ ↑↑ $f(x)=2(x+3)^1(x)^2(x-2)^1$		-3 Linear	0 Quad	2 Linear	∞	∞
$ax^n = 5x^7$ ↓↑ $f(x)=5(x+2)^1(x-1)^4(x-3)^2$		-2 Linear	1 Quad	3 Quad	$-\infty$	∞
$ax^n = -x^7$ ↑↓ $f(x)=-(x+3)^3(x-1)^2(x-2)^2$		-3 Cubic	1 Quad	2 Quad	∞	$-\infty$
$ax^n = -2x^4$ ↓↓ $f(x)=-2(x+4)^1(x+1)^1(x)^2$		-4 Linear	-1 Linear	0 Quad	$-\infty$	$-\infty$

1. Write the equation of the cubic function whose shown to the right.



f(x) = $\left(\frac{1}{3}(x+1)\right)^3+3$

Simplify

2. $(3x^5+1)+(9x^5+3x-2)$

$\boxed{12x^5+3x-1}$

3. $(3x^3+3x^2+9)-(5x^3-7x^2+6x-9)$

$-5x^3+7x^2-6x+9$

$\boxed{-2x^3+10x^2-6x+18}$

4. $(2x-5)(5x^2+4x+7)$

$10x^3+8x^2+14x$

$-25x^2-20x-35$

$\boxed{10x^3-17x^2-6x-35}$

5. $(3x-7)^2$

	$3x$	-7
$3x$	$9x^2$	$-21x$
-7	$-21x$	$+49$

$\boxed{9x^2-42x+49}$

Divide to determine if given binomial is a factor of f(x). If it is, factor f(x) completely, and then sketch the graph of f(x). (Disregard the y-scale)

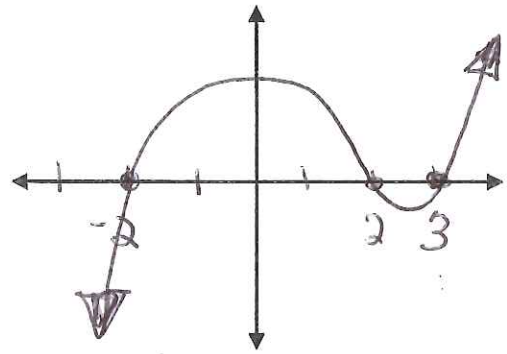
6. $f(x) = x^3 - 3x^2 - 4x + 12;\ (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

yes, it's a factor

$x^2 - x - 6$

$fx = \boxed{(x-3)(x+2)(x-2)}$



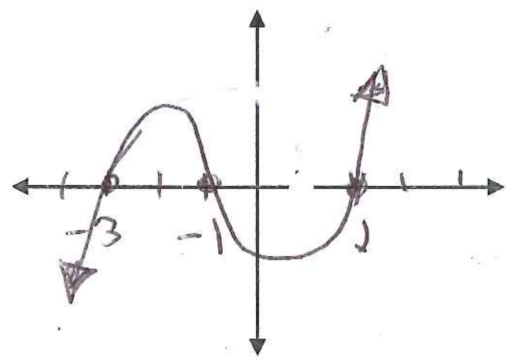
7. $f(x) = x^3 + 2x^2 - 5x - 6;\ (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

yes, it's a factor

$x^2 - x - 2$

$fx = \boxed{(x-2)(x+1)(x+3)}$



8. $f(x) = x^4 + 2x^3 - 12x^2 - 18x + 27;\ (x + 3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 2 & -12 & -18 & 27 \\ \hline & 1 & -1 & -9 & 9 & \boxed{0} \end{array}$$

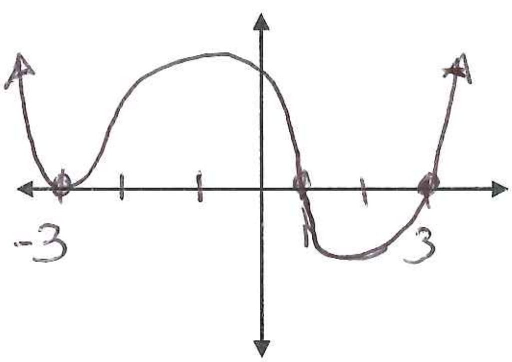
yes, it's a factor

$x^3 - x^2 - 9x + 9$ Grouping Method

$x^2(x-1) - 9(x-1)$

diff of squares $(x^2 - 9)(x-1)$ $\quad fx = \boxed{(x+3)^2(x-3)(x-1)}$

$(x+3)(x-3)$



9. $f(x) = x^3 - 6x^2 + 32;\ (x + 2)$

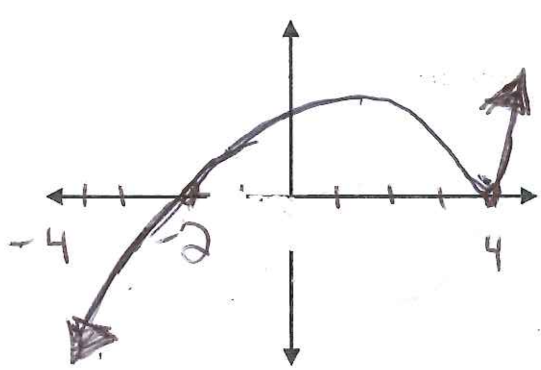
$$\begin{array}{r|rrrr} -2 & 1 & -6 & 0 & 32 \\ \hline & 1 & -8 & 16 & \boxed{0} \end{array}$$

yes, it's a factor

$x^2 - 8x + 16$ perf. square

$(x-4)^2$

$\boxed{f(x) = (x-4)^2(x+2)}$



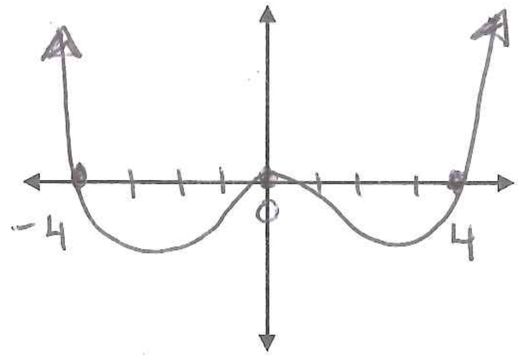
Factor each polynomial function completely, and sketch its graph. (Disregard the y-scale)

10. $f(x) = x^4 - 16x^2$ ↑↑

$x^2(x^2-16)$

$(x+4)(x-4)$

$\boxed{f(x) = x^2(x+4)(x-4)}$



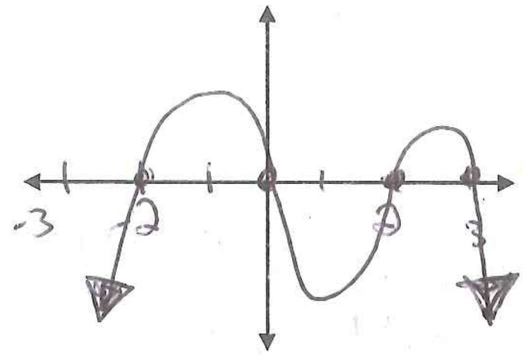
11. $f(x) = -x^4 + 3x^3 + 4x^2 - 12x$ ↓↓

$-x(x^3-3x^2-4x+12)$

$-x[x^2(x-3)-4(x-3)]$

$-x(x^2-4)(x-3)$

$\boxed{f(x) = -x(x+2)(x-2)(x-3)}$



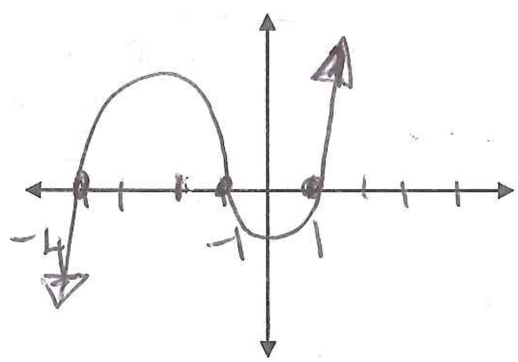
12. $f(x) = x^3 + 4x^2 - x - 4$ ↓↑

$x^2(x+4) - 1(x+4)$

$(x^2-1)(x+4)$

$(x+1)\ (x-1)$

$\boxed{f(x) = (x+1)(x-1)(x+4)}$



For Questions 13-18 Perform the operation using Long Division on the even number problems and Synthetic Division on the odd number problems.

13. $(x^3 - 3x^2 - 108x) \div (x - 12)$

12	1	-3	-108	0
		12	108	0
	1	9	0	0

$\boxed{x^2+9x}$

14. $(4x^2 - 9) \div (2x + 3)$

$$\begin{array}{r} 2x - 3 \\ 2x+3 \enclose{longdiv}{4x^2 + 0x - 9} \\ -(4x^2 + 6x) \\ \hline -6x - 9 \\ -(-6x - 9) \\ \hline 0 \end{array}$$

$\boxed{2x - 3}$

15. $(x^4 - 20x^3 + 98x^2 + 20x - 99) \div (x - 11)$

$$\begin{array}{c|ccccc} 11 & 1 & -20 & 98 & 20 & -99 \\ & & 11 & -99 & -11 & 99 \\ \hline & 1 & -9 & -1 & 9 & \boxed{0} \end{array}$$

$\boxed{x^3 - 9x^2 - x + 9}$

16. $(x^4 + 3x^3 - 43x^2 - 75x + 450) \div (x - 3)$

$$\begin{array}{r} x^3 + 6x^2 - 25x - 150 \\ x-3 \enclose{longdiv}{x^4 + 3x^3 - 43x^2 - 75x + 450} \\ -x^4 + 3x^3 \\ \hline 6x^3 - 43x^2 \\ -6x^3 + 18x^2 \\ \hline -25x^2 - 75x \\ +25x^2 + 75x \\ \hline -150x + 450 \\ +150x + 450 \\ \hline 0 \end{array}$$

$\boxed{x^3 + 6x^2 - 25x - 150}$

17. $(6x^2 + x - 2) \div (2x - 1)$ $\div 2$

$$\begin{array}{c|ccc} \frac{1}{2} & 6 & 1 & -2 \\ & & 3 & 2 \\ \hline & 6 & 4 & \boxed{0} \end{array}$$

$\frac{6}{2}$ $\frac{4}{2}$

$3x + 2$

$\boxed{3x + 2}$

18. $(-3x^3 - 66x^2 - 360x) \div (x + 10)$

$$\begin{array}{r} -3x^2 - 36x \\ x+10 \enclose{longdiv}{-3x^3 - 66x^2 - 360x} \\ +3x^3 + 30x^2 \\ \hline -36x^2 - 360x \\ +36x^2 + 360x \\ \hline 0 \end{array}$$

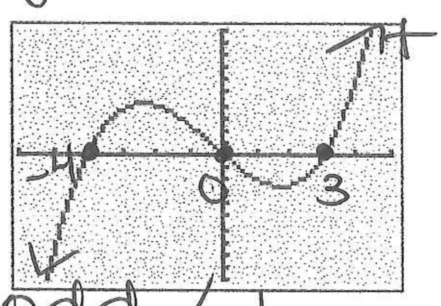
$\boxed{-3x^2 - 36x}$

Describe the cubic transformations:

19. $f(x) = -4(x-7)^3 - 3$ - Reflected over x-axis - Vertical stretch (by 4) - Right 7 units - Down 3 units	20. $y = \left(\frac{1}{3}(x-5)\right)^3 + 3$ - Horizontal Stretch (by 3) - Right 5 units - Up 3 units

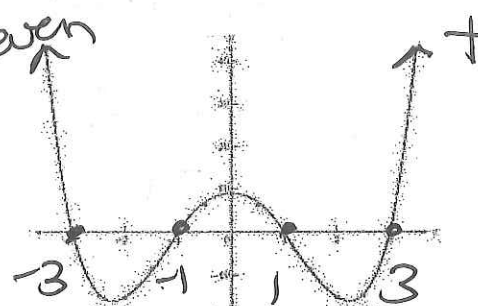
#21 – 23 : State if this is an even or odd degree polynomial and whether or not the leading term is positive or negative. Write the equation of the graph, with the least possible degree, in factored form.

21. Eq: $y = x(x+4)(x-3)$



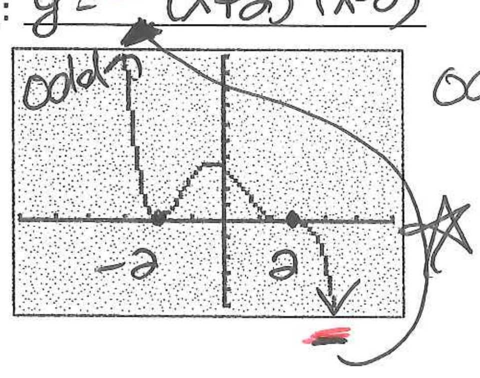
odd / +

22. Eq: $y = (x+3)(x+1)(x-1)(x-3)$



even / +

23. Eq: $y = -(x+2)^2(x-2)^3$



odd / −

24. Write a polynomial function with integer coefficients in **Standard** form using the given information. $(x+4)(x^2-5)$ $y = -\frac{1}{2}(x+4)(x^2-5)$

A cubic equation with roots at -4, $-\sqrt{5}, \sqrt{5}$ and a constant term of 10

y-int: (0,10)

Equation: $y = -\frac{1}{2}$

Method 1: constants: $10 = a(4)(-5)$ $10 = -20a$ $a = -\frac{1}{2}$

Method 2: y-int: $10 = a(0+4)(0^2-5)$ $10 = -20a$ $a = -\frac{1}{2}$

Method 3: Standard form: $y = a(x+4)(x^2-5)$ $y = a(x^3+4x^2-5x-20)$ $-20 \cdot -\frac{1}{2}$ $+10$

Factor:

25. $16n^3 - 250$

GCF: $2(8n^3 - 125)$

$\boxed{2(2n-5)(4n^2+10n+25)}$

for signs

S. O. A. P

Same

Opposite

Always

Positive

$$\underbrace{27x^6}_{a^3} + \underbrace{8y^3}_{b^3}$$

26. $27x^6 + 8y^3$

$a = 3x^2$

$b = 2y$

$\boxed{(3x^2+2y)(9x^4-6x^2y+4y^2)}$